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FREE VIBRATION CHARACTERISTICS OF LAMINATED COMPOSITE JOINED CONICAL-CYLINDRICAL SHELLS

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1. INTRODUCTION

Structural elements such as cylindrical, conical, and joined conical–cylindrical shells are commonly used as principal structural elements in the design of aerospace and nuclear structures. Furthermore, the structural sections, in particular, joined conical–cylindrical shells are extensively used as hydraulic nozzles, diffusers, horn antennae, etc. Information about the dynamic behavior of such joined conical–cylindrical shells with different system parameters may become advantageous for the economical designs.

The study of free vibration behaviours of isotopic conical and cylindrical shells has been carried out by many investigators and is well documented by Leissa [1]. Limited work on the study of vibrations of isotropic joined conical-cylindrical shells has been carried out by Hu and Raney [2], Lashkari and Weingarten [3] and Irie *et al.* [4]. Theoretical study using transverse matrix analysis has been employed in reference [4] whereas experimental and analytical studies have been made in references [2, 3].

The work concerning the characteristics of laminated anisotropic circular cylindrical shells has been recently reviewed in the work of Noor and Burton [5]. Some of the important contributions related to vibration of composite shells are cited here. Shivakumar and Krisnamurty [6] have dealt with the vibration of laminated circular cylindrical shells, whereas vibration and damping of laminated composite shells have been analyzed by Alam and Asnani [7]. The dynamic study of laminated conical shell has been carried out recently by Ramesh and Ganesan [8], Khatri and Asnani [9] and Korjakin *et al.* [10]. Although many investigators have developed general finite-difference/finite element or numerical integration techniques that allow the evaluation of dynamic behaviours of the arbitrary laminated composite shells of revolution, the available studies in the literature are mostly limited to either conical or cylindrical shells. However, no specific discussion has been found in the literature concerning dynamic analysis, in particular, free vibration characteristics of the laminated fibre-reinforced composite shells whose meridian contains a geometric discontinuity (conical–cylindrical and conical–cylindrical–conical shells). This has prompted the present authors to study and provide data for the joined laminated composite shells reported here.

For the purpose of this study, the finite element method is used for analysing the free vibration of laminated anisotropic composite conical–cylindrical shell structures. A simple two-noded shear flexible axi-symmetric shell element based on field consistency approach [11] is employed. The in-plane and rotary inertia effects are included in the model. The formulation developed here is used for obtaining the natural frequencies and the mode

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shapes for the laminated cross-ply composite joined conical–cylindrical and conical–cylindrical–conical shell systems.

2. FORMULATION

An axi-symmetric laminated composite joined conical–cylindrical–conical shell is considered with the co-ordinates s, θ along the meridional and circumferential directions, and z along the radial or thickness direction, as shown in Figure 1. By using the Mindlin formulation, the displacements at a points (s, θ, z) are expressed as functions of the mid-plane displacements u_0, v_0 and w, and independent rotations β_s and β_θ of the meridional and hoof sections respectively, as

$$u(s, \theta, z, t) = u_0(s, \theta, t) - z\beta_s(s, \theta, t),$$

$$v(s, \theta, z, t) = v_0(s, \theta, t) - z\beta_\theta(s, \theta, t)$$

$$w(s, \theta, z, t) = w(s, \theta, t) \tag{1}$$

where t is the time.

By using the semi-analytical approach, u_0, v_0, w, β_s, and β_θ are represented by a Fourier series in the circumferential angle θ. For the nth harmonic, these can be written as

$$u_0 = u_0(s, t)\cos n\theta, \quad v_0 = v_0(s, t)\sin n\theta, \quad w = w(s, t)\cos n\theta,$$

$$\beta_s = \beta_s(s, t)\cos n\theta, \quad \beta_\theta = \beta_\theta(s, t)\sin n\theta. \tag{2}$$

The strains pertaining to a truncated cone are defined as [12]

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_p \\ 0 \end{Bmatrix} + \begin{Bmatrix} -z\varepsilon_b \\ \varepsilon_s \end{Bmatrix}. \tag{3a}$$

The mid-plane strains ε_p, bending strains ε_b and shear strains ε_s, in equation (3a) are written as

$$\{\varepsilon_p\} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial s} \\ \left(u_0 \sin\phi + \dfrac{\partial v_0}{\partial \theta} + w\cos\phi\right)\Big/ r \\ \left(\dfrac{\partial u_0}{\partial \theta} - v_0 \sin\phi\right)\Big/ r + \partial v_0/\partial s \end{Bmatrix}, \tag{3b}$$

$$\{\varepsilon_b\} = \begin{Bmatrix} -\,\partial\beta_s/\partial s \\ -\,(\beta_s \sin\phi + \partial\beta_\theta/\partial\theta)/r \\ (\cos\phi\; \partial v_0/\partial s - \partial\beta_s/\partial\theta + \beta_\theta \sin\phi)/r - \partial\beta_\theta/\partial s \end{Bmatrix}, \tag{3c}$$

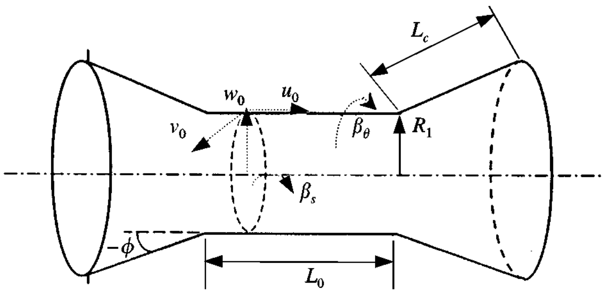

Figure 1. Geometry of a joined conical–cylindrical–conical shell (three-section).

$$\{\varepsilon_s\} = \begin{Bmatrix} \dfrac{\partial w}{\partial s} - \beta_s \\ \dfrac{\dfrac{\partial w}{\partial \theta} - v_0 \cos\phi}{r} - \beta_\theta \end{Bmatrix}, \tag{3d}$$

where $r(s)$ is equal to $R_1 + s\sin\phi$ and the subscript comma denotes the partial derivative with respect to the spatial co-ordinate succeeding it. R_1 and ϕ are the small circle radius and semi-cone angle respectively.

IF $\{N\}$ represents the membrane stress resultants (N_{ss}, $N_{\theta\theta}$, $N_{s\theta}$) and $\{M\}$ the bending stress resultants (M_{ss}, $M_{\theta\theta}$, $M_{s\theta}$), one can relate these to the membrane strains $\{\varepsilon_p\}$ and bending strains $\{\varepsilon_b\}$ through the constitutive relation as

$$\{N\} = [A]\{\varepsilon_p\} + [B]\{\varepsilon_b\} \quad \text{and} \quad \{M\} = [B]\{\varepsilon_p\} + [D]\{\varepsilon_b\}, \tag{4}$$

where $\lfloor A_{ij} \rfloor$, $\lfloor B_{ij} \rfloor$, $\lfloor D_{ij} \rfloor$ $(i, j = 1, 2, 6)$ are the extensional, bending-extensional coupling, and bending stiffness coefficients of the composite laminate. Similarly, the transverse shear force $[Q]$ representing the quantities (Q_{sz}, $Q_{\theta z}$) are related to the transverse shear strains $\{\varepsilon_s\}$ through the constitutive relation as

$$\{Q\} = [E]\{\varepsilon_s\}, \tag{5}$$

where $[E_{ij}]$ $(i, j = 4, 5)$ are the transverse shear stiffness coefficients of the laminates. For a composite laminate of thickness h, consisting of l layers with stacking angles α_i $(i = 1, \ldots, l)$ and the layer thickness h_i $(i = 1, \ldots, l)$, the necessary expression to compute the stiffness coefficients, available in the literature [13] are used here.

The strain energy functional U is given as

$$U(\delta) = (1/2)\int_A [\{\varepsilon_p\}^{\mathrm{T}}[A]\{\varepsilon_p\} + \{\varepsilon_p\}^{\mathrm{T}}[B]\{\varepsilon_b\} + \{\varepsilon_b\}^{\mathrm{T}}[B]\{\varepsilon_p\} + \{\varepsilon_b\}^{\mathrm{T}}[D]\{\varepsilon_b\} + \{\varepsilon_s\}^{\mathrm{T}}[E]\{\varepsilon_s\}]\,\mathrm{d}A, \tag{6}$$

where $\{\delta\}$ is the vector of the degrees of freedom.

The kinetic energy of the shell is given by

$$T(\delta) = (1/2)\int_A [p(\dot{u}_0^2 + \dot{v}_0^2 + \dot{w}^2) + I(\dot{\beta}_s^2 + \dot{\beta}_\theta^2)]\,\mathrm{d}A, \tag{7}$$

where $p = \int_{-h/2}^{h/2} \rho \, dz$, $I = \int_{-h/2}^{h/2} \rho z^2 \, dz$ and ρ is the mass density. A dot over the variable represents the partial derivative with respect to time.

Substituting Eqs. (6) and (7) into Lagrange's equation of motion, one obtains the governing equation for free vibrations of the shell as

$$[M]\{\ddot{\delta}\} + [K]\{\delta\} = \{0\}, \tag{8}$$

where $[M]$ and $[K]$ are the mass matrix and stiffness matrices respectively.

The eigenvalues and the associated mode shapes are evaluated using the standard eigenvalue extraction algorithm.

3. RESULTS AND DISCUSSION

In the section, we use the above formulation to investigate the effects of parameters like cone angle and number of cross-ply layers on the free vibration behavior of laminated joined conical-cylindrical and conical-cylindrical-conical shells. Since the finite element is based on consistency approach, exact integration is used to evaluate all the energy terms. The shear correction factor is taken as 5/6. Based on progressive mesh refinement, a 40-element idealization for two-section shell geometry, and a 60-element idealization for a three-section case are found to be well adequate to model the full shell for the present analysis. Firstly, the formulation developed herein is validated considering free vibration analysis of isotropic joined conical–cylindrical shell [4] and laminated circular sandwich conical [10, 14] and the results are compared in Tables 1 and 2 along with the available solutions. It is observed from these tables that the present results are in fairly good agreement with those of the existing results. The small discrepancies in the results may be attributed to the different theories and solution approaches used in the literature [4, 10, 14]. The material properties of CFRP, unless specified otherwise, used in the present analysis are

$$E_L/E_T = 39{\cdot}8113, \quad G_{LT}/E_T = 0{\cdot}4906, \quad G_{TT}/E_T = 0{\cdot}2453, \quad \nu_{LT} = 0{\cdot}25,$$

$$\rho = 1524\ \mathrm{kg/m^3}, \quad E_T = 0.053 \times 10^{11}\ \mathrm{N/m^2},$$

where E, G, and ν are Young's modulus, shear modulus and Poisson's ratio. L and T are the longitudinal and transverse directions respectively, with respect to fibres. All the layers are

TABLE 1

Comparison of natural frequencies of a cantilever cylindrical–conical isotropic shell: ($L_c/R_1 = 2$, $L_0/R_1 = 1$, $h/R_1 = 0{\cdot}01$, $\nu = 0{\cdot}3$, $\phi = 30°$)

	Circumferential wave number (n); f (rad/s)					
	Reference [4]			Present		
$m^{\dagger}$	0	1	5	0	1	5
1	0·5047	0·293	0·2021	0·5016	0·2618	0·2061
2	0·9312	0·6363	0·2966	—	0·607	0·2962
3	0·9566	0·8116	0·373	0·953	0·8161	—
4	0·9718	0·9316	0·5805	0·963	—	0·5961
5	1·0122	0·9528	0·6138	0·984	0·9634	0·6215

†Axial half wave number.

TABLE 2

Natural frequencies $\bar{f}(Hz)$ *for the clamped–clamped conical sandwich shell*: (*top/bottom layer*: $E_f = 25{\cdot}08$ *GPa*, $\nu_f = 0{\cdot}20$, $\rho_f = 2800$ *kg/m*^{3}, $h_f = 0{\cdot}535$ *mm*; *core layer*: $G_{12} = 0{\cdot}2204$ *GPa*, $G_{13} = 0{\cdot}126$ *GPa*, $\rho_c = 36{\cdot}8$ *Kg/m*^{3}, $h_c = 7{\cdot}62$ *mm*; $Lc = 1{\cdot}8415$ *m*, *small radius* = $0{\cdot}5702$ *m*, $\phi = 5{\cdot}07°$)

m	n	Present	Reference [14]	Reference [10]
1	1	304·52	—	300·4
	2	181·72	177·20	184·7
	3	127·28	126·00	127·9
	4	111·65	110·00	111·6
	5	127·95	126·70	128·4
	6	165·05	163·50	166·4
2	1	519·78	—	520·5
	2	342·35	340·10	349·8
	3	255·85	254·30	250·4
	4	210·50	209·70	200·6
	5	199·22	197·70	188·5
	6	216·78	214·8	209·7

of equal thickness and the ply-angle is measured with respect to the s-axis (longitudinal axis). The boundary conditions considered in the present analysis are:
simply supported

$$u = w = \beta_\theta = 0 \quad \text{at } s = 0, L$$

clamped–clamped

$$u = v = w = \beta_s = \beta_\theta = 0 \quad \text{at } s = 0, L.$$

Here, the free vibration analysis of laminated conical–cylindrical shells ($L_c/R_1 = (\operatorname{cosec}\phi + \sec\phi)/2$; $L_0/R_0 = 1$; $R_1/h = 300$ where L_c and L_0 are the meridional length of conical and cylindrical section and R_0 is the radius of cylinder) is carried out for fundamental/lowest mode, i.e., n corresponding to n_{cr}, n denotes the circumferential full wave number. The plots of non-dimensional frequency $\Omega(=\bar{\omega}\sqrt{\rho_0 h R_1^4/D}$; $D = E_L h^3/[12(1-\nu_{LT}\nu_{TL})]$) with the respect to cone angles for different combinations of cone ($\phi = 0$, 15, 30 and 45°) and cylinder sections are depicted, for simply supported and clamped boundary conditions. The conical section assumed at the ends of the cylindrical shell is of either convergent or divergent type (with respect to cylinder).

The variations of fundamental frequency and its associated wave number n_{cr} for cross-ply laminates are highlighted in Figure 2 for simply supported joined conical-cylindrical shells (two section). It is seen from Figure 2(a) that, in general, for the cylindrical shell with convergent conical section, critical wave number n_{cr} corresponding to the fundamental frequency is less in comparison with those of cylindrical one with divergent conical end case. Also, it is brought out from Figure 2(b) that the increase in the cone angle, irrespective of the type of cone (convergent or divergent), enhances the value of natural frequency. This is mainly due to the increase in the severity of the geometric discontinuity that, in turn, produces a stiffening effect around the joint (cone–cylinder). Furthermore, it is noticed from Figure 2(b) that the value of frequency is much higher for the convergent conical–cylindrical shell compared to those of divergent conical–cylindrical case. However, for the cylindrical

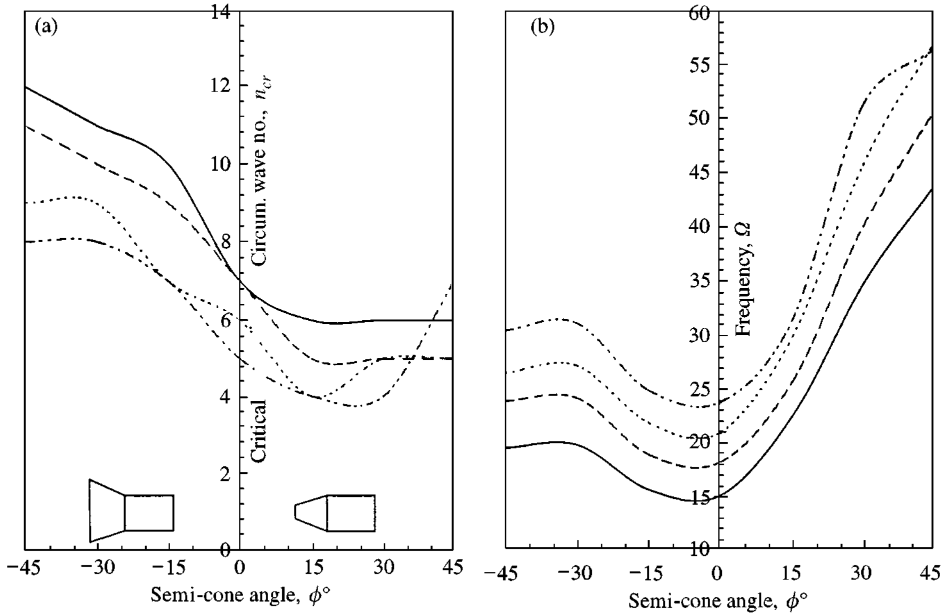

Figure 2. Effect of semi-cone angle of laminated composite convergent/divergent conical–cylindrical shells with simply supported ends on free vibrations (two-section): (a) critical circumferential wave number, (b) fundamental frequency at n_{cr}. (——) 0°; (...........) 0°/90°; (– – – –), 0°/90°/0°; (–···–··), (0°/90°/0°/90°)$_s$.

shell with divergent conical end structure, the differences in the values of the frequency are insignificant for the higher cone angles considered here.

Also, one can note from Figure 2 that the influence of the number of layers in the cross-ply laminated conical–cylindrical shells on the dynamic behavior is significant. In general, the single-layered orthotropic shell section has the highest critical value for circumferential wave number n, and then followed by the three-layered, two-layered cases, and lastly, by the eight-layered symmetric laminate, except at higher cone angle for the convergent conical–cylindrical case. For the convergent conical–cylindrical shell with higher cone-angle, the critical wave number increases for two-layered and eight-layered shells. For the chosen cone section at the end of the cylindrical shell, the variation in the value of n_{cr} and its associated frequency highly depend on the directional stiffness provided by the anisotropic properties in the laminates. It is further seen from Figure 2(b) that the frequency is very low for the single-layered orthotropic shell, and next higher cases are corresponding to the three-layered, two-layered, and finally the eight-layered symmetric laminates. The behaviours such as variations of n_{cr} and, in turn, change in the values of frequency highly depend on the contribution of membrane, bending and to some extent shear strain energies to the total internal energy of the shell structure. For instance, a higher n_{cr} value for orthotropic case, in general, may lead to very less contribution of membrane energy to the total strain energy and thus, the system has very low value of the flexural frequency. Also, it is brought out from Figure 2(b) that, for the cylindrical section with convergent conical end, the frequency values are higher than those of the cylindrical section with divergent cone. Further, the presence of coupling effect due to two-layered laminate is to lower the value of frequency compared to the case of eight-layered one.

Free vibration study is also made for joined conical–cylindrical–conical shell (three-section). Keeping the same cylindrical shell geometry as that of two-section and attaching conical section at both ends of the cylindrical shell, three-section shell geometry is

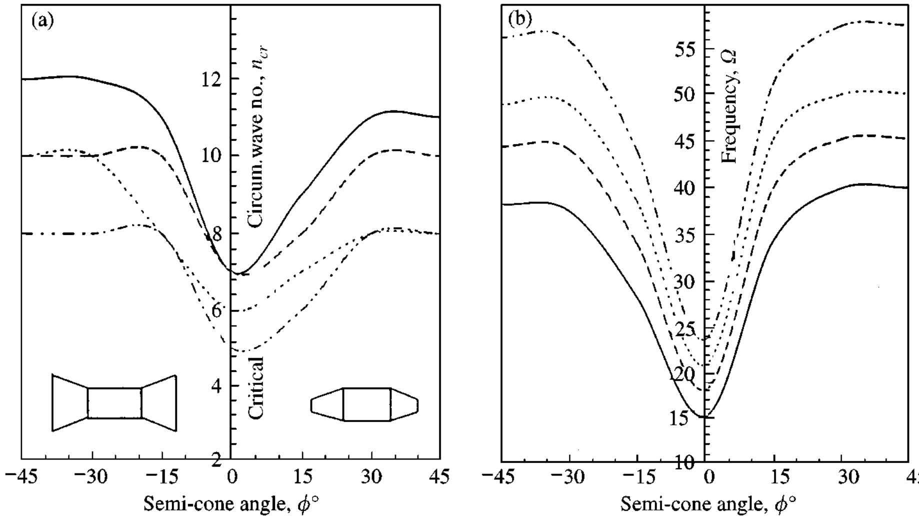

Figure 3. Effect of semi-cone angle of laminated composite convergent/divergent conical–cylindrical shells with simply supported ends on free vibrations (three-section): (a) critical circumferential wave number, (b) fundamental frequency at n_{cr}. ——, 0°; ...., 0°/90°; – – – –, 0°/90°/0°; – · · – · ·, $(0°/90°/0°/90°)_s$.

considered. The length of each conical section is assumed as half of the conical portion pertaining to the two-section cases. The dynamic characteristics of laminated shells predicted here are described in Figure 3 for the simply supported case.

It is understood from Figure 3(a) that, for the three-section shell, the critical circumferential wave number for the convergent conical shell combinations, irrespective of the cone angles, is considerably high compared to those of the two-section case whereas it is slightly more for the divergent conical shell combinations. However, for the divergent conical shell case having higher cone angle, there appears to be no change in the values of n_{cr} while comparing with the two-section one (Figure 2(a)). Furthermore, it is observed from Figure 3(b) that, for the cylindrical shell with divergent conical ends, the value of frequency is very high compared to the value of the two-section case. It can be also noticed that, for the case of cylindrical shell with convergent conical ends compared to the corresponding two-section case, the change in the value of natural frequency is less for the higher cone angle, and more so for the deep conical sections.

It is clear from Figure 3(a) that the influence of number of layers on the n_{cr} is qualitatively similar to those of two-section shells. However, one can notice here that the value of n_{cr} increases with the increase in the divergent cone angle, up to a certain value. But unlike in the case of two-section shells, here, n_{cr} increases with the cylindeical shell having convergent conical end sections. It can be also opined from Figure 3(b) that the variation in the trend of frequency curve is almost symmetrical with respect to cone angles.

A similar study is carried out considering clamped ends for the two-section and three-section cases, and their dynamic characteristics are exhibited in Figures 4 and 5, respectively. The behaviours of the shell with respect to n_{cr} and the variation in the frequency trend are qualitatively very similar to those of the simply supported one. It is observed from these figures that the increase in the values of the natural frequency corresponding to n_{cr} is very less for the clamped boundary condition considered here in comparing with those of simply supported shell.

The mode shapes for the selected geometries and chosen circumferential wave numbers are presented for both simply supported and clamped shells having two-section in Figures 6

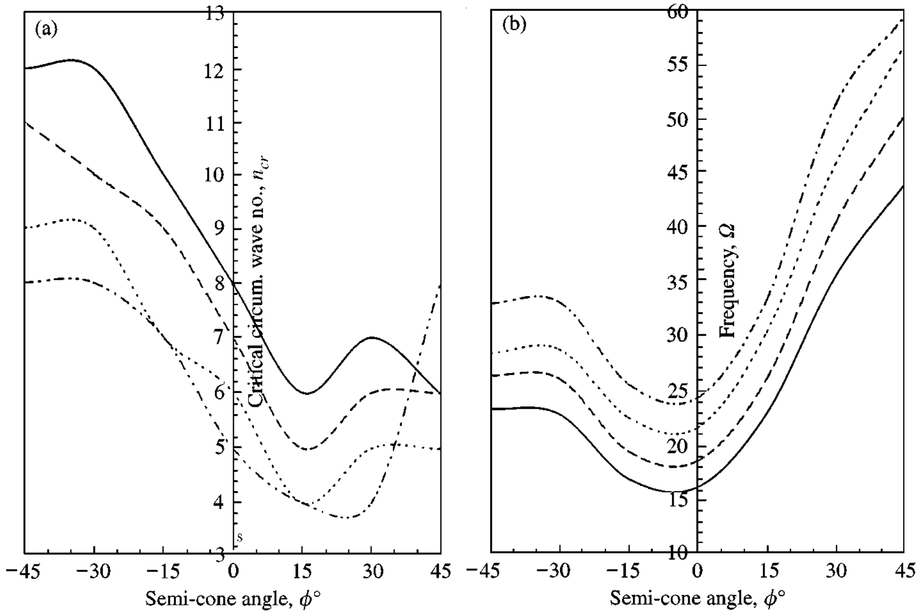

Figure 4. Effect of semi-cone angle of laminated composite convergent/divergent conical–cylindrical shells with clamped ends on free vibrations (two-section): (a) critical circumferential wave number, (b) fundamental frequency at n_{cr}. ——, 0°; ...., 0°/90°; – – – –, 0°/90°/0°; – · · – · ·, $(0°/90°/0°/90°)_s$.

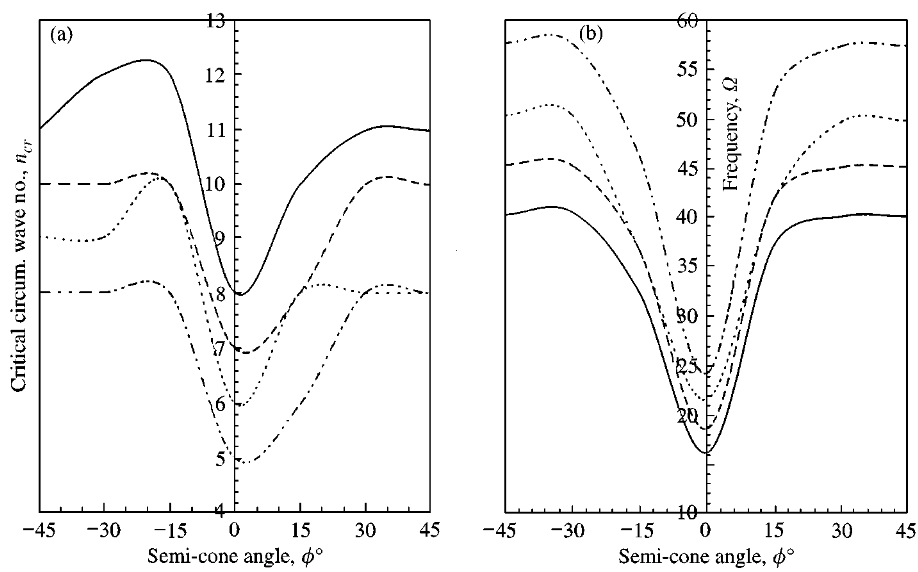

Figure 5. Effect of semi-cone angle of laminated composite convergent/divergent conical–cylindrical–conical shells with clamped ends on free vibrations (three-section): (a) critical circumferential wave number, (b) fundamental frequency at n_{cr}. ——, 0°; ...., 0°/90°; – – – –, 0°/90°/0°; – · · – · ·, $(0°/90°/0°/90°)_s$.

and 7. The meridional mode shapes of the normalised normal displacement W, in-plane displacements U and V are highlighted in these figures. It is noticed from Figures 6 and 7 that there is a sharp change in the mode shapes at the junction of cone–cylinder shell, irrespective of the cone angle considered here and, in turn, indicates the stiffening effect of

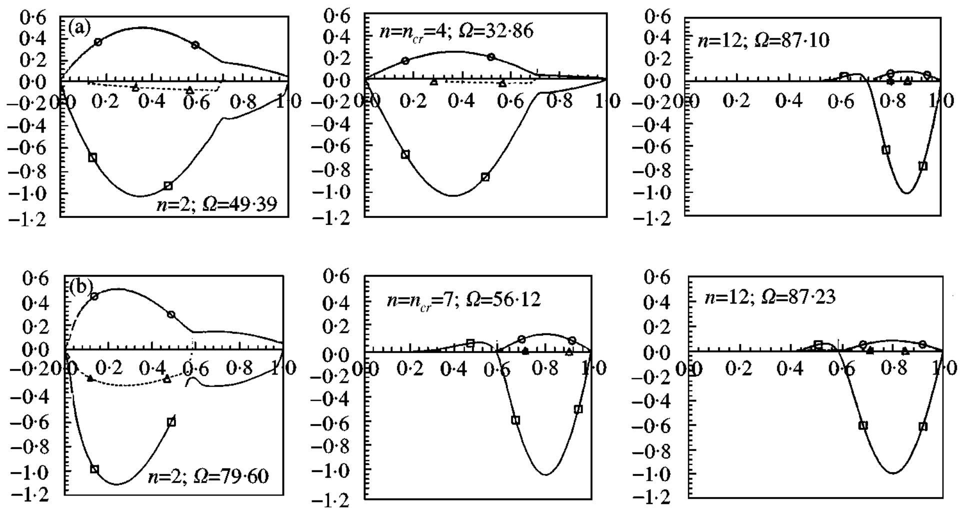

Figure 6. Mode shapes of simply supported two-section shells with different conical sections and circumferential wave numbers ($n = 2$, n_{cr} & 12): (a) $\phi = 15°$, (b) $\phi = 45°$ (–⊟– W; –⊖– V; ·····△····· U).

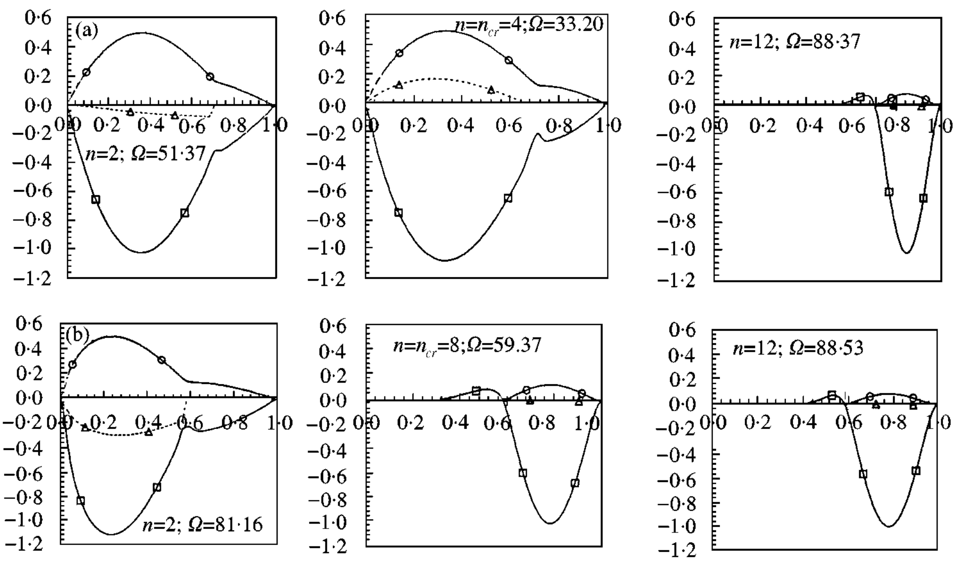

Figure 7. Mode shapes of clamped two-section shells with different conical sections and circumferential wave numbers ($n = 2$, n_{cr} & 12): (a) $\phi = 15°$, (b) $\phi = 45°$ (–⊟– W; –⊖– V; ·····△····· U).

the joint and the presence of the localised high bending stresses. It is also observed that the change in the displacement gradient is more for W. One can also conclude from these figures that the in-plane motions decrease with the increase in the value of circumferential wave number. However, normal displacement, W highly depends on n and cone angle. It is further inferred that for low value of n, the joint acts as an elastic support for the conical component of the shell having low cone-angle whereas it is like a hinged support for the cylinder when the number is more than n_{cr}. Furthermore, it exhibits that the deflection pattern of V is similar to that of W, but is in the opposite direction. For higher cone-angle case, the cone portions acting as hinged supports are seen even at $n = n_{cr}$, irrespective of the type of end supports. For the low value of n, U and V are more significant for the clamped case compared to those of the simply supported one.

Similar mode shape plots for the three-section case concerning clamped supports are shown in Figure 8 for the selected cone sections (convergent or divergent with 15° and 45°) and circumferential wave number, n (2, n_{cr} and 12). One can notice from Figures 8(a) and 8(c) that the change in the gradient of the displacement is very high for the three-section case compared to those of the two-section case. For cylindrical shell with convergent cone

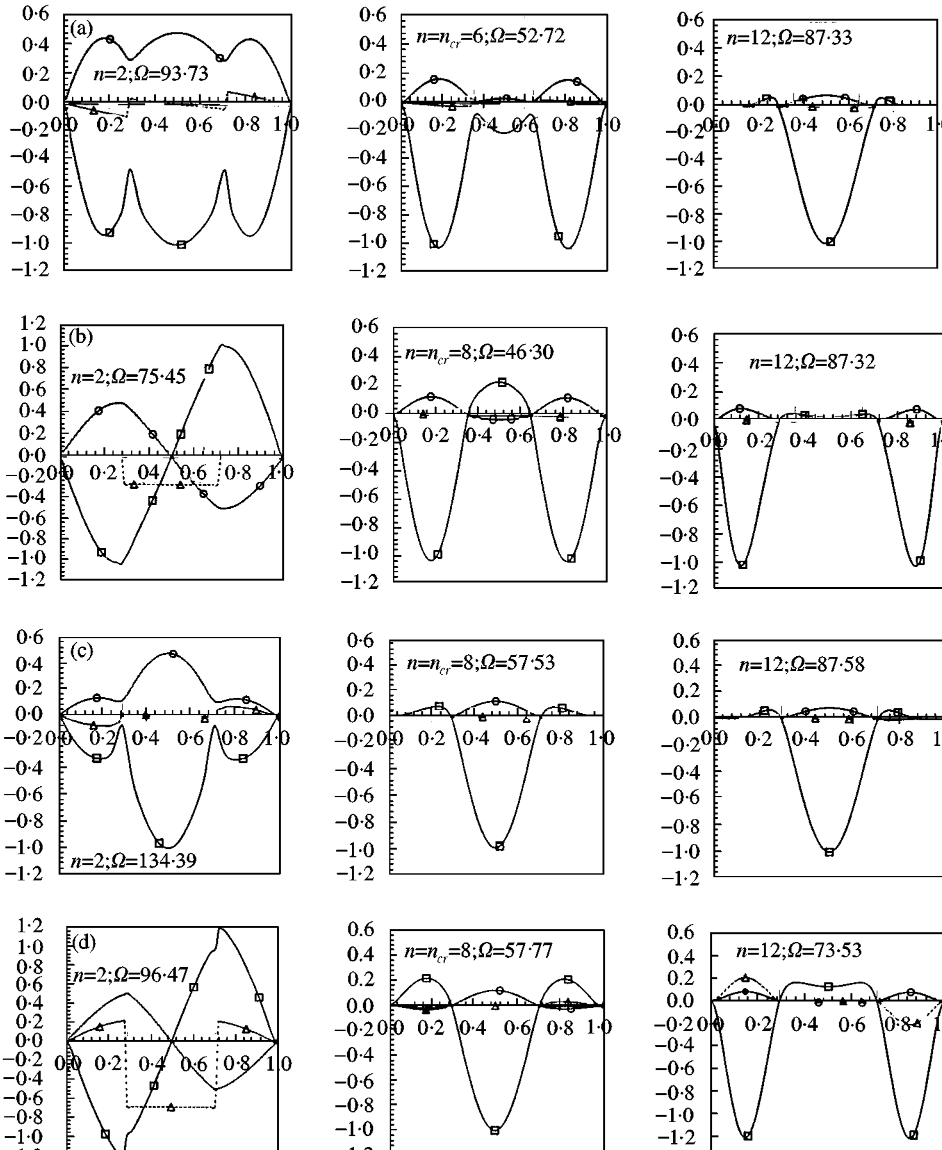

Figure 8. Mode shapes of clamped three-section shells with different conical sections and circumferential wave numbers ($n = 2$, n_{cr} & 12): (a) $\phi = 15°$, (b) $\phi = -15°$, (c) $\phi = 45°$, (d) $\phi = -45°$ (—□— W; —○— V; ·····△····· U).

sections, the joints act as hinged supports for the cylinder, especially when the shell has large value of *n*, and shows insignificant values for the displacements over the conical sections, as observed in the two-section case. The mode shapes for the divergent cases are given in Figure 8(b) with 15°, and in Figure 8(d) with 45° cone angle. For a low value of *n*, the variation of the normal displacement *W* and in-plane displacement *V* is linear over the cylindrical component whereas it is constant and increases with cone angle for a large value of *n*. It is further opined from Figures 8(b) and 8(d) that, in general, the conical parts undergo high displacement compared to the cylindrical section and it depends on the value of *n* and divergent cone angle. It is hoped that this study will be useful for the designers/engineers while dealing with shell structures having geometric discontinuity under dynamic situations.

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